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Multiple anomalous U(1)s in heterotic blow-ups

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Abstract

The existence of multiple anomalous U(1)s is demonstrated explicitly in a blow-up version of a heterotic \mathbb{Z}_3 orbifold. Another blow-up of the same orbifold supports further evidence for the type-I/heterotic duality in four dimensions. It has a single anomalous U(1) which does not factorize universally. As multiple anomalous U(1)s as well as non-universal factorization have never been established on heterotic orbifolds explicitly, these findings might appear contradictory at first sight. Possible inconsistencies are avoided by reinterpreting a charged twisted state as a second non-universal localized axion. The mismatch between the charges of the orbifold and blow-up spectra is resolved by suitable field redefinitions. The anomaly of the field redefinitions corresponds to the difference of blow-up and heterotic orbifold anomalies.

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1 Introduction and results

For phenomenological applications of heterotic string compactifications it is important to know the number of anomalous $U(1)$ s [1, 2]. It has been generally accepted that heterotic orbifold models [3–6] contain at most a single anomalous $U(1)$. Recent publications [7, 8] argued that the low energy limit of the heterotic string, i.e. super Yang–Mills (YM) theory coupled to supergravity, on general smooth Calabi–Yau (CY)s with $U(1)$ bundle backgrounds multiple anomalous $U(1)$ s are possible. Assuming that such CYs have continuous singular orbifold limits, the existence of multiple anomalous $U(1)$ s seems puzzling. In 6D matching between heterotic orbifolds and blow–ups seems always possible [9]. To investigate this issue in 4D, explicit blow–ups of the orbifold fixed points would be very instructive. Since for the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold the blow–up and its $U(1)$ bundles have recently been constructed [10] (see also [11]), we would like to focus on anomalous $U(1)$ s in such blow–ups.

The following obstructions seem to prevent a straightforward identification of heterotic orbifold models and their blow–up counterparts:

1. In contrast to heterotic orbifolds, blow–ups can have more than one anomalous $U(1)$.
2. While orbifold theories seem to contain at most a single axion relevant for Green–Schwarz (GS) anomaly cancellation, blow–up models can have multiple axions.
3. In blow–up models with a single anomalous $U(1)$ the anomalies do not factorize universally as they do in heterotic orbifold models.
4. The spectra of orbifolds and their corresponding blow–ups often do not match.
5. Even when the non–Abelian spectra agree, the $U(1)$ charges of twisted states are never identical.

We argue that all these discrepancies can be explained by suitable field redefinitions. In particular, we reinterpret a charged twisted singlet on the orbifold, that drives the blow–up by taking Vacuum Expectation Value (VEV), as a localized axion. It takes part in a non–universal GS anomaly cancellation, very much like twisted RR–axions in type–I models. The anomaly in these field redefinitions precisely corresponds to the difference between blow–up and heterotic orbifold anomalies.

We first describe this procedure for $\mathbb{C}^3/\mathbb{Z}_3$ blow–up models with $U(1)$ bundles in general. After that we inspect two concrete blow–up models of a specific heterotic \mathbb{Z}_3 orbifold to illustrate some details. The orbifold model has been considered before [12, 13] as evidence for heterotic/type–I duality [14] in 4D. The second example constitutes a different blow–up of the same orbifold that has two anomalous $U(1)$ s.

2 Multiple Anomalous $U(1)$ s in Blow–Up

In [10] the blow–up \mathcal{M}^3 of the $\mathbb{C}^3/\mathbb{Z}_3$ orbifold and its line bundles are described. Using these results we decompose the 10D GS 2–form B_2 into 4D perturbations as

$$B_2 = b_2 + i\mathcal{F}_2 b_0 + \omega_2 B_0 . \quad (1)$$

Both the $U(1)$ bundle field strength $i\mathcal{F}_2$ and the Kähler form ω_2 are harmonic 2–forms. (For the Kähler form this follows automatically, while for the $U(1)$ field strength this is guaranteed by the

Hermitean YM equations.) Aside from the 4D anti-symmetric tensor b_2 , dual to the universal axion a^{uni} , the 2-form B_2 is decomposed into two 4D scalars b_0 and B_0 . The B_0 is interpreted as the 6D internal B_2 , i.e. the nine untwisted $B_{\underline{i}j}$, $\underline{i}, j = 1, 2, 3$, because ω_2 reduces to the orbifold Kähler form in the blow down limit. The state b_0 is localized near the orbifold singularity, since $i\mathcal{F}$ becomes strongly peaked there in that limit [10]. The expansion (1) is complete: As each of the nine untwisted $B_{\underline{i}j}$ states contributes $1/27$ at a \mathbb{Z}_3 fixed point [15], the blow-up has $h_{11} = 9/27 + 1 = 4/3$, which indeed equals half its Euler number [10].

Which of the b_2 , b_0 and B_0 mediate GS mechanisms is determined by their anomalous variations, that leave the 3-form field strength H_3 of B_2 invariant. Expanding H_3 in these 4D perturbations gives

$$H_3 = db_2 - \Omega_3^{YM} + \Omega_3^L + i\mathcal{F}_2(db_0 - iA_V) + \omega_2 dB_0. \quad (2)$$

The usual anomalous variations of b_2 are found by restricting the YM and gravitational Chern-Simons (CS) 3-forms, Ω_3^{YM} and Ω_3^L , to 4D. Since the expansion of 10D YM CS also contains a linear term in $i\mathcal{F} H_V$, the anomalous variation of b_0 is determined by $iA_V = \text{tr}[H_V iA_1]$ with $H_V = V^I H_I$. The H_I are the Cartan generators of 10D gauge group and V^I are numbers characterizing the U(1) bundle [10]. Contrary B_0 never has an anomalous variation, because neither CS 3-forms have decompositions proportional to ω_2 .

The 4D anomaly I_6 is obtained from the factorized 10D anomaly polynomial $I_{12} = X_4 X_8$, where $X_4 = \text{tr}(iF_2)^2 - \text{tr} R^2$ and X_8 is given in [16, 17]. By integrating over the blow-up we get

$$I_6 = \int_{\mathcal{M}^3} I_{12} = \int_{\mathcal{M}^3} X_{2,2} X_{4,4} + X_{0,4} X_{6,2}, \quad (3)$$

where the first (second) index specifies the number of 6D internal (4D Minkowski) indices of the forms. A third possible contribution, $X_{4,0} X_{2,6}$, integrates to zero due to the background Bianchi identity. The integral of the first term can give rise to more than one term in general. However, for the U(1) bundles on the blow-up of $\mathbb{C}^3/\mathbb{Z}_3$ we always get

$$X_{2,2} = 2 i\mathcal{F}_2 \text{tr}(H_V iF_2), \quad (4)$$

which ensure that this gives a single factor. Factorization in heterotic orbifolds only occurs through the last term in (3). Hence the blow-up of $\mathbb{C}^3/\mathbb{Z}_3$ can support at most two anomalous U(1)s, and we can interpret $b_0 = a^{non}$ as a non-universal axion. Together with the universal axion a^{uni} (4D dual to b_2) it will take part in the GS mechanism of anomaly cancellation.

3 Spectral matching

Above we argued that on the blow-up of $\mathbb{C}^3/\mathbb{Z}_3$ orbifold there can be two anomalous U(1)s. In this section we explain why there is no contradiction with the general finding that heterotic orbifolds have at most a single anomalous U(1). Consider a heterotic orbifold model and a proposal for a corresponding blow-up. Because the heterotic orbifold has an anomalous U(1), the vacuum is unstable and some twisted fields need to get non-vanishing VEVs. Depending on which fields attain VEVs this is accompanied by further symmetry breaking to the blow-up gauge group. Some twisted fields decouple by VEV induced superpotential mass terms; this makes the non-Abelian spectrum of the heterotic orbifold equal to that of the blow-up model [12, 13]. But this does not resolve why charges of the fields present on both sides can be different.

A possible mismatch of U(1) charges can be understood by analyzing the consequences of fields with non-vanishing VEVs. Let Ψ_q be a twisted singlet chiral superfield, w.r.t. the unbroken gauge group with U(1) charges $q = (q_1, \dots, q_n)$, and Φ_Q another twisted chiral superfield, not necessarily a singlet, with charges Q . A non-vanishing VEV v of Ψ_q means that in the quantum theory this field can be represented as an exponential, and we can redefine Φ_Q with an arbitrary power r of this exponential:

$$\Psi_q = e^T v, \quad \Phi_Q = e^{rT} \Phi_{Q'} . \quad (5)$$

The new superfields T and $\Phi_{Q'}$ transform under the U(1) gauge transformations as:

$$T \rightarrow T + iq_i \varphi_i, \quad \Phi_{Q'} \rightarrow e^{i(Q_i - r q_i) \varphi_i} \Phi_{Q'}, \quad (6)$$

where φ_i are the gauge parameters of the U(1)s. Hence the imaginary part a_T of T is related to the non-universal axion a^{non} of the blow-up theory. In addition, we have obtained a superfield $\Phi_{Q'}$ with U(1) charges $Q' = Q - r q$. We claim that it is always possible to perform such field redefinitions to make the charges of the non-decoupled twisted states equal to their blow-up counterparts. Moreover, the fields that decouple can be given gauge invariant superpotential mass terms using field redefinitions.

Next we explain how these field redefinitions help to get agreement between the anomalies of the heterotic orbifold and the blow-up models. Upto this point a_T only has anomalous gauge variations (6) but no anomalous couplings; still at this stage only one axion is involved in the GS mechanism. We call this the heterotic axion a^{het} . Its anomalous couplings are determined by the anomaly polynomial, that factorizes universally $I_6^{het} = X_2^{het} X_4$. Here the GS 4-form X_4 is restricted to 4D, and X_2^{het} , via the descent equations, determines the anomalous variation of a^{het} . The anomalous couplings for the axion a_T arises because the path integral measure is not invariant under the anomalous field redefinitions (5). The resulting couplings can be deduced from the anomaly polynomial I_6^{red} associated with the field redefinitions. It factorizes as $I_6^{red} = q_i i F_2^i X_4^{red}$, where $i F_2^i$ denote the U(1) field strengths, because all field redefinitions (5) involve only T . This anomaly combined with the heterotic anomaly I_6^{het} equals the anomaly of the blow-up model:

$$I_6^{het} + I_6^{red} = I_6^{blo} = I_6^{uni} + I_6^{non} . \quad (7)$$

Hence after the field redefinitions the chiral spectra of the heterotic orbifold and the blow-up models become identical.

Finally we would like to understand the precise relation between the heterotic axion a^{het} and the axion a_T obtained from the superfield T used in the rescaling (5), and the two blow-up axions, a^{uni} (the 4D dual of $b_{\mu\nu}$) and $a^{non} = b_0$. The anomaly polynomials I_6^{uni} and I_6^{non} of the latter two are determined by the anomalous couplings of the zero modes of B_2

$$\int B_2 X_8 = \int b_2 X_{2,6} + (b_0 i \mathcal{F}_2 + B_0 \omega_2) X_{4,4} . \quad (8)$$

They factorize as

$$(2\pi)^2 I_6^{uni} = X_2^{uni} X_4, \quad (2\pi)^2 I_6^{non} = X_2^{non} X_4^{non}, \quad (9)$$

where $X_2^{non} = -\frac{1}{96} \text{tr} [H_V i F_2]$ and

$$X_2^{uni} = \int_{\mathcal{M}^3} \frac{X_{2,6}}{96(2\pi i)^3}, \quad X_4^{non} = -\int_{\mathcal{M}^3} \frac{2i \mathcal{F}_2 X_{4,4}}{(2\pi i)^3} . \quad (10)$$

Model	Gauge Group	Spectrum
Het.O.	$\text{SO}(8) \times \text{U}(12)$	$\frac{1}{9}(\mathbf{8}, \mathbf{12})_1 + \frac{1}{9}(\mathbf{1}, \overline{\mathbf{66}})_{-2} + (\mathbf{1}, \mathbf{1})_4 + (\mathbf{8}_+, \mathbf{1})_{-2}$
TypeI	$\text{SO}(8) \times \text{U}(12)$	$\frac{1}{9}(\mathbf{8}, \mathbf{12})_1 + \frac{1}{9}(\mathbf{1}, \overline{\mathbf{66}})_{-2}$
$\text{U}(1)^2$	$\text{U}(4) \times \text{U}(12)$	$\frac{1}{9}(\mathbf{4}, \mathbf{12})_{1,1} + \frac{1}{9}(\overline{\mathbf{4}}, \mathbf{12})_{1,-1} + \frac{1}{9}(\mathbf{1}, \overline{\mathbf{66}})_{-2,0} + (\mathbf{6}, \mathbf{1})_{0,2}$

Table 1: The spectra of the heterotic \mathbb{Z}_3 orbifold and two blow-ups are displayed: The spectrum of “TypeI” equals the type-I \mathbb{Z}_3 orbifold, and “ $\text{U}(1)^2$ ” has two anomalous $\text{U}(1)$ s.

Hence each term in (7) can be computed independently, providing a consistency check. The relation between the axions is now fixed by noting that the anomalous variations of the axions a^{het} , a_T , a^{uni} and a^{non} are determined from X_2^{het} , $q_i iF_2^i$, X_2^{uni} and X_2^{non} , respectively, via the standard descent equations. Therefore, the sum relation for the anomalies (7) implies that the interactions of the axions with gauge and gravitational fields are related via

$$a^{het} X_4 + a_T X_4^{red} = a^{uni} X_4 + a^{non} X_4^{non} . \quad (11)$$

As the unbroken gauge group typically consists of two or more group factors, this leads to an over constrained system of linear equations relating (a^{uni}, a^{non}) to (a^{het}, a_T) . Because both a^{het} and a^{non} multiply X_4 , we find

$$a^{uni} = a^{het} + c a_T , \quad a^{non} = d a_T , \quad (12)$$

where c and d are model dependent constants.

To summarize we have shown that the chiral spectra of the heterotic orbifold and blow-up models upon using field redefinitions that allow one to modify $\text{U}(1)$ charges. In particular, a charged twisted singlet on the heterotic orbifold is reinterpreted as a localized axion in the blow-up theory. The remaining spectra have a modified anomaly structure, which is precisely canceled by the anomalous variation of this localized axion.

4 A heterotic $\text{SO}(32)$ \mathbb{Z}_3 orbifold

We illustrate our general findings by considering two $\text{U}(1)$ bundle blow-ups of the heterotic $\text{SO}(32)$ $\mathbb{C}^3/\mathbb{Z}_3$ orbifold model with gauge shift $\frac{1}{3}(0^4, 1^8, -2^4)$. The gauge group and the spectrum of this heterotic model [18–20] are given in the first row of table 1. Since a multiplicity factor of $\frac{1}{9}$ signals untwisted modes, the model contains two twisted states: a singlet $(\mathbf{1}, \mathbf{1})_4$ and a $\text{SO}(8)$ spinor $(\mathbf{8}_+, \mathbf{1})_{-2}$ with charges 4 and -2, respectively. As usual for heterotic models the anomaly polynomial factorizes universally

$$(2\pi)^2 I_6^{het} = -\frac{1}{3} iF_1 X_4 , \quad (13)$$

$$X_4 = 24(iF_1)^2 + 2\text{tr}(iF_{12})^2 + \text{tr}(iF_8)^2 - \text{tr} R^2 .$$

The blow-ups are determined by VEVs of the twisted states that satisfy the D- and F-flatness conditions. The D-flatness implies that at least the singlet has a VEV to cancel the one-loop Fayet–Iliopoulos term [1,2]. (In addition, if the spinor takes a VEV, two of its components are non-vanishing.)

Assuming that none of the untwisted states have VEVs, the relevant part of the superpotential reads

$$W \sim (\mathbf{1}, \mathbf{1})_4 [(\mathbf{8}_+, \mathbf{1})_{-2}]^2 . \quad (14)$$

(When both $(\mathbf{1}, \mathbf{1})_4$ and $(\mathbf{8}_+, \mathbf{1})_{-2}$ have VEVs, F-flatness requires higher order superpotential terms.)

This heterotic orbifold model has received quite some attention in the past because this model was suggested to have a type-I \mathbb{Z}_3 orbifold model dual [12, 13, 21, 22]. However, because the GS anomaly cancellation in both models is mediated by different fields, it had been questioned whether these models can really be dual [23] to each other. Applying the general formalism we developed here, the duality is realized in all fine print.

4.1 Type-I blow-up model

If we only give a VEV to $(\mathbf{1}, \mathbf{1})_4$, the gauge group of the blow-up model remains the same as on the heterotic orbifold. We identify this case with blow-up model characterized by $V = (0^4, 1^{12})$ defined in ref. [10]. The spectra of the blow-up model and the type-I \mathbb{Z}_3 orbifold model [21, 22] are identical, and given in the second row of table 1. Even though there is just a single $U(1)$, it does not factorize universally:

$$(2\pi)^2 I_6^{blo} = -\frac{iF_1}{96} \left(12(iF_1)^2 + \text{tr}(iF_{12})^2 - \text{tr}(iF_8)^2 - \frac{1}{8} \text{tr} R^2 \right).$$

Because only the singlet takes a VEV, we make the following field redefinitions of the twisted states

$$(\mathbf{1}, \mathbf{1})_4 = e^T v , \quad (\mathbf{8}_+, \mathbf{1})_{-2} = e^{-\frac{1}{2}T} (\mathbf{8}_+, \mathbf{1})'_0 . \quad (15)$$

The superpotential (14) gives the state $(\mathbf{8}_+, \mathbf{1})'_0$ a regular mass. This transformation takes a twisted singlet to a localized axion in the blow-up theory; from the type-I point of view this is the twisted RR -axion. By computing the various anomaly polynomials we derive the identification of the axions

$$a^{non} = -\frac{1}{16} a_T , \quad a^{uni} = a^{het} + \frac{1}{8} a_T . \quad (16)$$

Hence the type-I \mathbb{Z}_3 orbifold model coincides with a blow-up of a heterotic \mathbb{Z}_3 orbifold.

4.2 Blow-up model with two anomalous $U(1)$ s

The second blow-up of the heterotic orbifold model is obtained by giving VEVs to both $(\mathbf{8}_+, \mathbf{1})_{-2}$ and $(\mathbf{1}, \mathbf{1})_4$. This induces further symmetry breaking of $SO(8)$ to $U(4)$, and therefore this model has two $U(1)$ s. The corresponding $\mathbb{C}^3/\mathbb{Z}_3$ blow-up model has a $U(1)$ bundle characterized by $V = -\frac{1}{2}(q + q')$ with the charge vectors $q = (1^{12}, 0^4)$ and $q' = (0^{12}, 1^4)$. The spectrum is given in the last row of table 1. The anomaly polynomial fails to factorize:

$$(2\pi)^2 I_6^{blo} = -\frac{2iF_1}{3} \left(12(iF_1)^2 - 8(iF'_1)^2 - 2\text{tr}(iF_4)^2 + \right. \\ \left. \text{tr}(iF_{12})^2 - \frac{1}{8} \text{tr} R^2 \right) + 2iF'_1 \left(4(iF'_1)^2 + \text{tr}(iF_4)^2 - \frac{1}{8} \text{tr} R^2 \right),$$

hence there are two anomalous $U(1)$ s. We can perform field redefinitions to match the spectra of the orbifold and its blow-up version up to two singlets. To this end we realize that the singlets $(\mathbf{1}, \mathbf{1})_{-2, -2}$ and $(\mathbf{1}, \mathbf{1})_{-2, 2}$ are obtained from $(\mathbf{8}_+, \mathbf{1})_{-2}$ after symmetry breaking. The redefined twisted states are:

$$(\mathbf{1}, \mathbf{1})_{-2, -2} = e^T v, \quad (\mathbf{1}, \mathbf{1})_{-2, 2} = e^{-T} (\mathbf{1}, \mathbf{1})'_{-4, 0}. \quad (17)$$

The singlets $(\mathbf{1}, \mathbf{1})_{4, 0}$, $(\mathbf{1}, \mathbf{1})'_{-4, 0}$ missing in the blow-up pair up to become massive. Also $(\mathbf{6}, \mathbf{1})_{-2, 0}$ gets mass terms because of Yukawa interactions involving $(\mathbf{1}, \mathbf{1})_{4, 0}$ that has a VEV as well. The relation between the axions is given by:

$$a^{non} = -\frac{1}{16} a_T, \quad a^{uni} = a^{het} - \frac{1}{16} a_T. \quad (18)$$

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